CHAPTER -6

TRIANGLES

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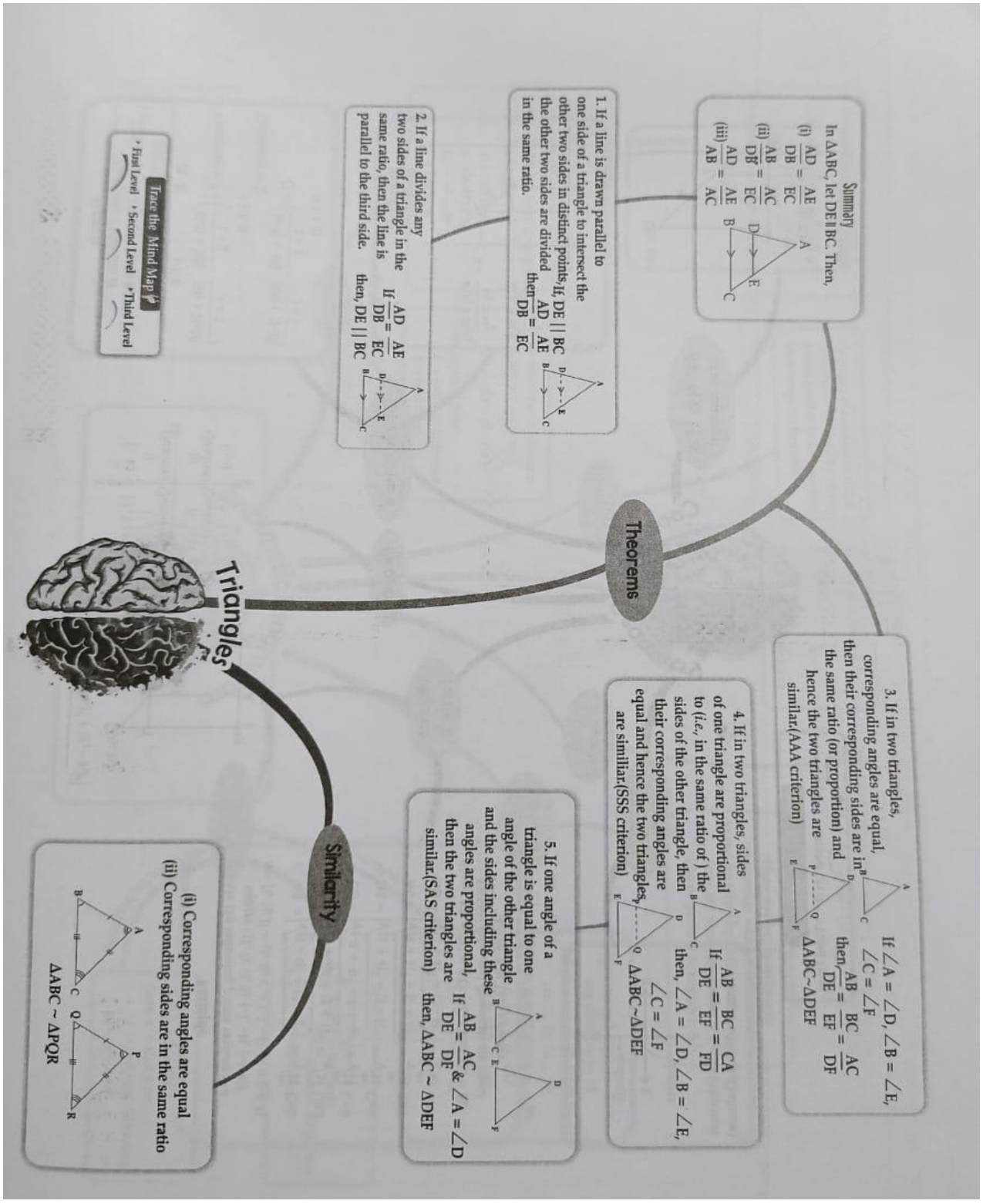
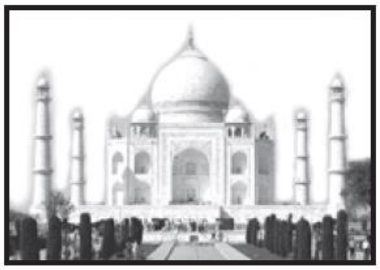
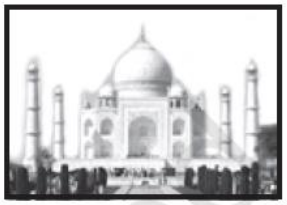
MIND MAP



CONCEPT:

- Two figures having the same shape but not necessarily the same size are called ***similar*** figures.

- Two figures are said to be ***congruent*** if they have the same shape and the same size

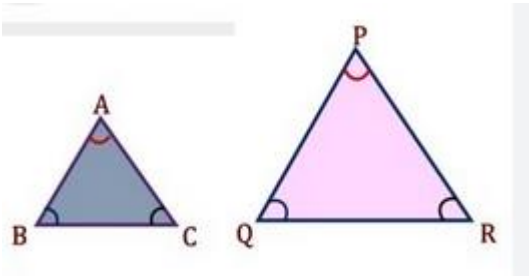

Note- *All congruent figures are similar but the similar figures need not be congruent.*

- Two ***polygons*** of the same number of sides are similar, ***if*** (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (i.e., proportion).
- **SIMILAR TRIANGLES**
-

Two triangles are said to be ***similar,*** if

(i) Their corresponding angles are equal and∠A=∠P, ∠B=∠Q, ∠C=∠R

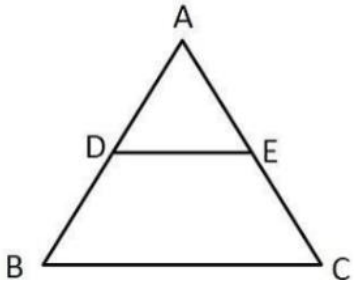
(ii) Their corresponding sides are in the same ratio (or proportion). AB/PQ =AC/PR = BC/QR



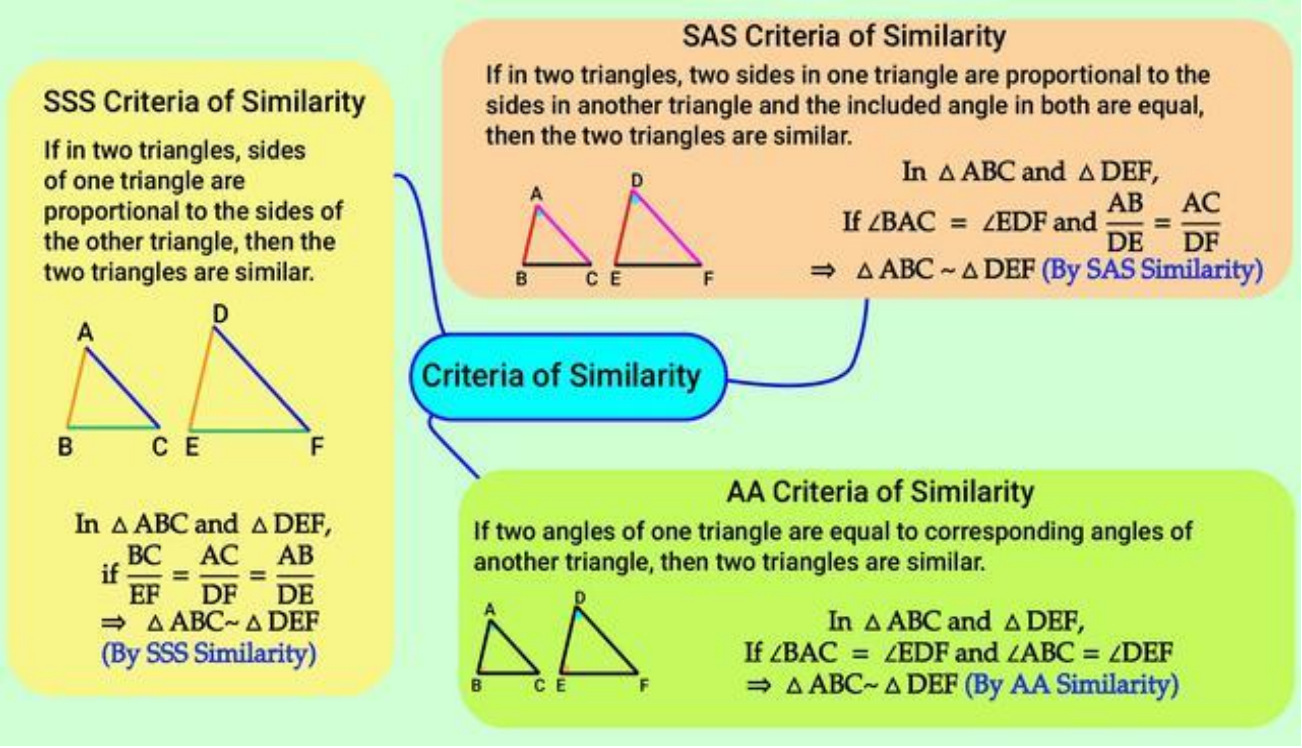
THEOREM:

Basic proportionality Theorem/ Thales Theorem: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

If DE// BC, Then $\frac{AD}{DB} = \frac{AE}{EC}$

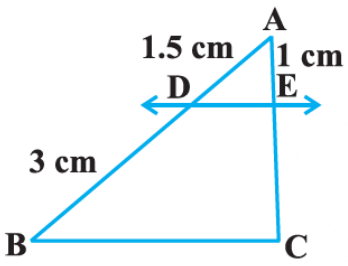


Criteria for similarity of triangles



NOTE: AA similarity criterion can also be stated as AAA similarity criterion.

SOLVED EXAMPLES



***Q1. If DE || BC. Find EC

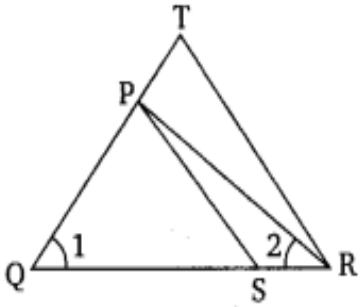
Ans- In ΔABC, DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ (By BPT)

$\frac{1.5}{3} = \frac{1}{EC}$

$EC = \frac{3}{1.5} = 2$ cm

**Q2. In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and ∠1 = ∠2. Show that ΔPQS ~ ΔTQR.



Ans- From the figure ∠1 = ∠2

PQ = PR (Sides opposite to equal angles are equal)

In Δ PQS and Δ TQR

Or $\frac{QR}{QS} = \frac{QT}{PR}$ (given)

Or $\frac{QR}{QS} = \frac{QT}{PQ}$ (PQ = PR)

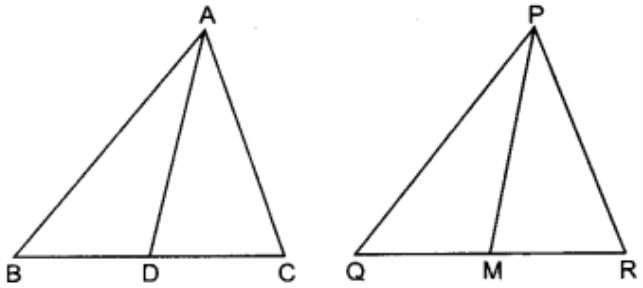
$\angle$ PQS = $\angle$TQR

$\therefore \Delta$ PQS ~ Δ TQR (By SAS)

***Q3. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$,**

prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Ans-



When $\Delta ABC \sim \Delta PQR$

$\Rightarrow \angle ABC = \angle PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$$

$$\frac{AB}{PQ} = \frac{BD}{QM}$$

In ΔABD and ΔPQM,

$$\frac{AB}{PQ} = \frac{BD}{QM}$$ [As proved]

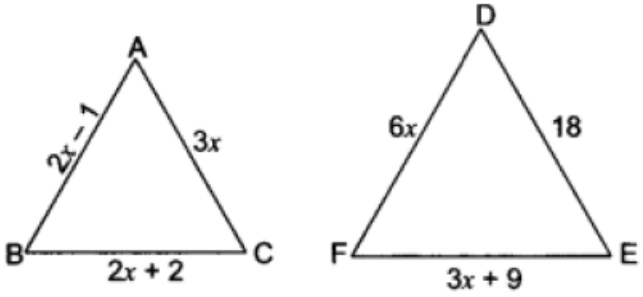
$\angle B = \angle Q$

$\therefore \Delta ABD \sim \Delta PQM$

$$\frac{AB}{PQ} = \frac{AD}{PM}$$ [Corresponding sides of similar triangles]

****Q4.**

In Fig, if $\Delta ABC \sim \Delta DEF$ and their sides are of lengths (in cm) as marked along with them, then find the lengths of the sides of each triangle



Ans- $\Delta ABC \sim \Delta DEF$ (Given)

therefore, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

So, $\frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$

Now, taking $\frac{2x-1}{18} = \frac{3x}{6x}$, we have

$$\frac{2x-1}{18} = \frac{1}{2}$$

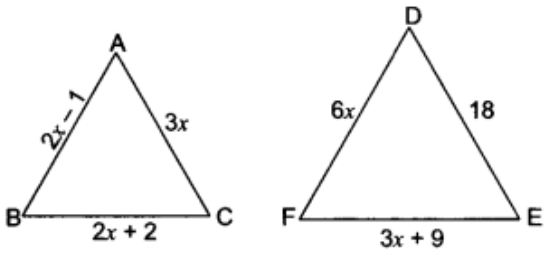


Fig. 7.26

$\Rightarrow 4x - 2 = 18$

$\Rightarrow x = 5$

$\therefore AB = 2 \times 5 - 1 = 9,$

$BC = 2 \times 5 + 2 = 12$

$CA = 3 \times 5 = 15,$

$DE = 18,$

$EF = 3 \times 5 + 9 = 24$ and

$FD = 6 \times 5 = 30$

Hence, AB = 9 cm, BC = 12 cm, CA = 15 cm DE = 18 cm, EF = 24 cm, FD = 30 cm

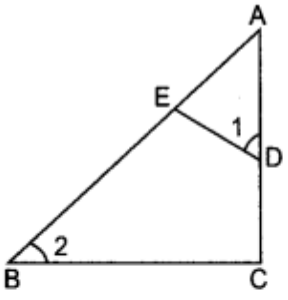
***Q5. In ΔABC, if ∠ADE = ∠B, then prove that ΔADE ~ ΔABC. Also, if AD = 7.6 cm, AE = 7.2 cm, BE = 4.2 cm and BC = 8.4 cm, then find DE.**

Ans-

Given: ∠ADE = ∠B, i.e. ∠1 = ∠2

To prove: ΔADE ~ ΔABC

Proof: In ΔADE and ΔABC



$\angle 1 = \angle 2$ [Given]

$\angle A = \angle A$ [Common]

So, $\Delta ADE \sim \Delta ABC$ [By AA similarity]

$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \frac{7.6}{7.2+4.2} = \frac{DE}{8.4}$ $\{\because AB = AE + BE = 7.2 + 4.2\}$

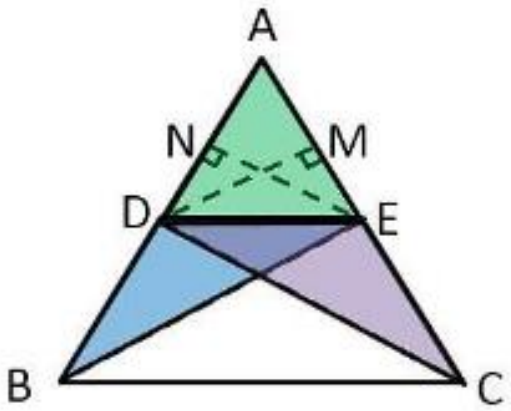
$\Rightarrow \frac{7.6}{11.4} = \frac{DE}{8.4} \Rightarrow DE = \frac{7.6 \times 8.4}{11.4} = 5.6$

Hence, DE = 5.6 cm.

*****Q6. State and prove Basic proportionality Theorem.**

Ans-

statement- ***If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.***



Given- A triangle ABC in which DE // BC

To prove- $\frac{AD}{DB}=\frac{AE}{EC}$

Construction: Join BE and CD and draw DM ⊥ AC and EN ⊥ AB.

Proof:

$$\text{ar (ADE)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times AD \times EN \quad \ldots(1)$$

$$\text{ar (BDE)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times DB \times EN \quad \ldots(2)$$

Divide (1) and (2)

$$\frac{\text{ar (ADE)}}{\text{ar (BDE)}} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN}$$

$$\frac{\text{ar (ADE)}}{\text{ar (BDE)}} = \frac{AD}{DB} \quad \ldots(A)$$

$$\text{ar (ADE)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times AE \times DM \quad \ldots(3)$$

$$\text{ar (DEC)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times EC \times DM \quad \ldots(4)$$

Divide (3) and (4)

$$\frac{\text{ar (ADE)}}{\text{ar (DEC)}} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM}$$

$$\frac{\text{ar (ADE)}}{\text{ar (DEC)}} = \frac{AE}{EC} \quad \ldots(B)$$

Since Δ BDE and ΔDEC are on the same base DE and between the same parallels BC and DE.

Therefore, ar (BDE) = ar (DEC)

Hence from (A), and (B), we have $\frac{AD}{DB}=\frac{AE}{EC}$ Proved.

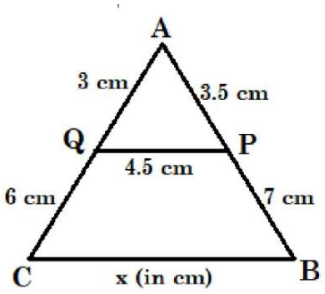
PRACTICE QUESTION

SECTION A (MULTIPLE CHOICE QUESTIONS 1 MARK)

***Q1.** Given that ΔABC~ΔDEF . If DE = 2AB and BC =3 cm then, EF is equal to _____.

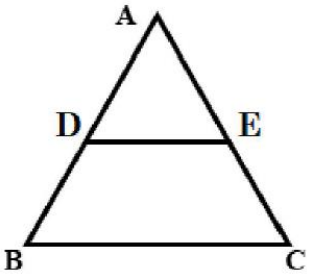
(a) 12 cm (b) 2 cm (c) 1.5 cm (d) 6 cm

****Q2**. In the given figure, write the value of x.



(a) 9 cm (b) 10.5 cm (c) 13.5 cm (d) 12 cm

***Q3**. In the given figure, $\frac{AD}{DB}=\frac{AE}{EC}$ and ∠ADE =70º, ∠BAC =50º, then ∠BCA =

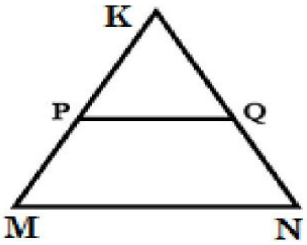


(a) 70º (b) 50º (c) 80º (d) 60º

***Q4.**The shadow of a tower 5 m long is 2 m . At the same time the shadow of a tree 12.5 m high is:

(a) 3 m (b) 3.5 m (c) 5 m (d) 4.5 m

****Q5.**In the figure given below, ,$\frac{KP}{PM}=\frac{4}{13}$, and KN = 20.4 cm, then the value of KQ is:



(a) 2.8 cm (b) 3.8 cm (c) 4.8 cm (d) 5.8 cm

****Q6. If** ΔPQR~ΔXYZ, ∠Q = 50º and ∠R = 70º then ∠X + ∠Y is equal to

(a) 70º (b) 50º (c) 120º (d) 110º

**Q7.If in triangles ABC and DEF, $\frac{AB}{DE}=\frac{BC}{FD}$, then they will be similar, when

(a) ∠B = ∠E (b) ∠A = ∠D (c) ∠B = ∠D (d) ∠A = ∠F

**Q8. In ΔABC, D is point on side AB and E is a point on side AC such that ∠ADE = ∠ABC, AD = 2, BD = 3 and AE = 3, then what is the value of CE?

(a) 6 cm (b) 3 cm (c) 4.5 cm (d) 5 cm

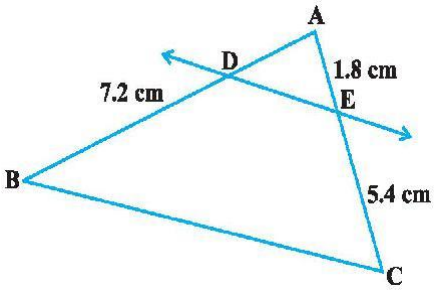
*Q9. If in two triangles ABC and DEF $\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$ then

(a) ΔFDE ~ ΔCAB (b) ΔFDE ~ ΔABC

(c) ΔCBA ~ ΔFDE (d) ΔBCA ~ ΔFDE

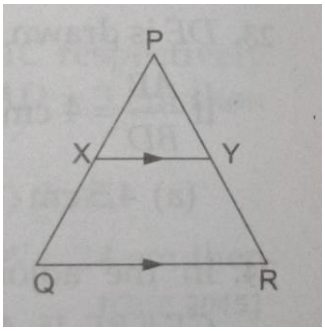
**Q10.In figure DE || BC then the value of AD is

(a) 2 cm (b) 2.4 cm (c) 3 cm (d) none of the above



***Q11. In the adjoining figure, XY // QR and PX : XQ =5:6. Then, XY : QR equals

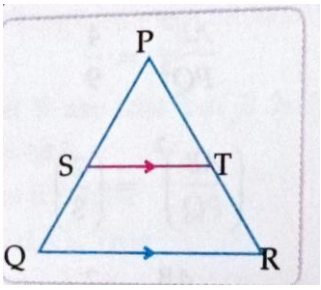
(a)5:11 (b) 6:5 (c) 11:5 (d)11:6



** Q12. In the following figure, ST//QR, Point S divides PQ in the ratio 4:.

If ST=1.6 cm, what is the length of QR?

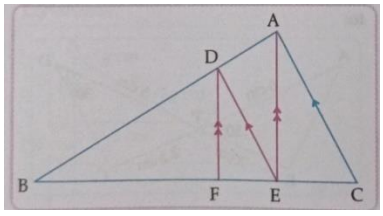
(a) 0.71 cm (b) 2 cm (c) 3.6 cm (d) none of the above



13.In the figure below, DE//AC and DF//AE.

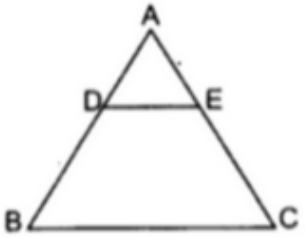
Which of these is equal to $\frac{BF}{FE}$?

(a) DF/AE
(b) BE/EC
(c) BA/AC



(d) FE/EC

****Q14.**In figure, if DE || BC, AD=3 cm, BD= 4 cm and BC= 14 cm, then DE equals



(a) 7 cm (b) 6 cm (c) 4 cm (d) 3 cm

****Q15.** D and E are the midpoints of side AB and AC of a triangle ABC, respectively and BC= 6cm.

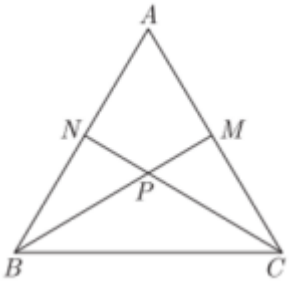
If DE || BC, then the length of DE is:

(a) 2.5 cm (b) 3cm (c)5cm (d) 6cm

SECTION B

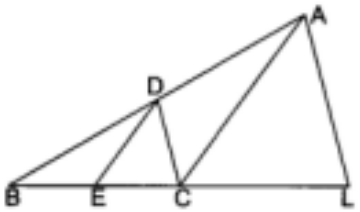
(SHORT ANSWER TYPE QUESTION 2 MARKS)

***Q16.** In the given figure, AM: MC =3:4, BP:PM =3:2 and BN = 12 cm. Then find AN



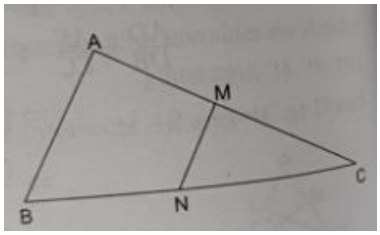
****Q17.**A street light bulb is fixed on a pole 12 m above the level of the street. If a woman of height 3 m casts a shadow of 6m, what is the length of the shadow of the pole?

****Q18**. In the given Fig, CD || LA and DE || AC. Find the length of CL, if BE = 4 cm and EC =2 cm.



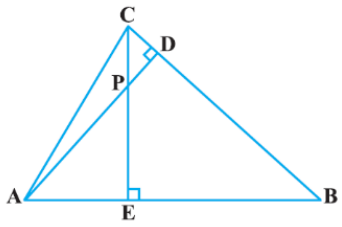
***Q19.** XY is drawn parallel to the base BC of a Δ ABC cutting AB at X and AC at Y. If AB = 4 BX and YC =2cm, then AY is

****Q20.**In the given figure, MN // AB, BC = 7.5cm, AM = 4cm and MC = 2cm. find the length of BN.



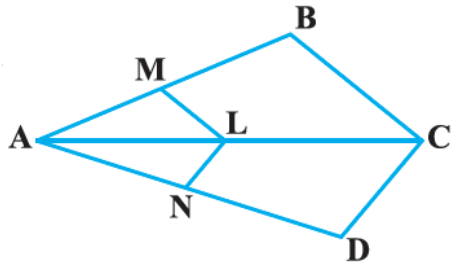
***Q21. In Fig., altitudes AD and CE of ΔABC intersect each other at the point P.

Show that: ΔAEP ~ ΔCDP



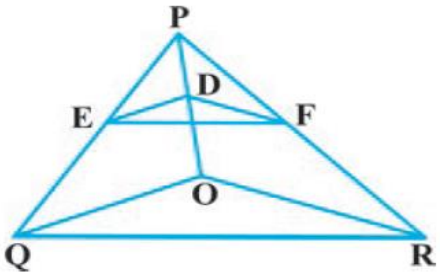
***Q22. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

***Q23.In the given figure, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$



*Q24.S and T are points on sides PR and QR of Δ PQR such that ∠ P = ∠ RTS.
Show that Δ RPQ ~ Δ RTS.

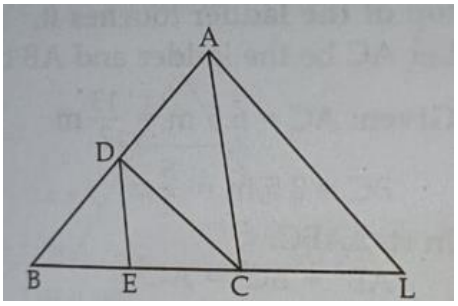
**Q25. In the given Figure, DE || OQ and DF || OR.
Show that EF || QR.



SECTION C

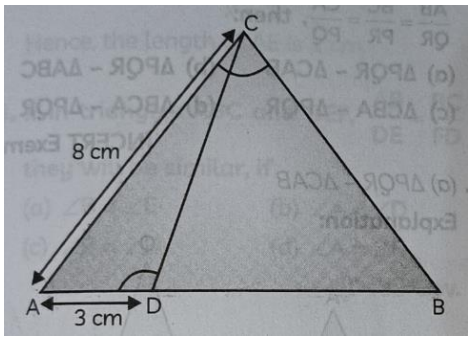
(SHORT ANSWER TYPE QUESTION 3 MARKS)

**Q26. In the given figure, CD // LA and DE // AC. Find the length of CL if BE = 4cm and EC = 2cm.

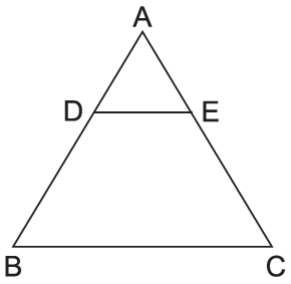


***Q27.** In a ΔABC, DE // BC with D on AB and E on AC. If $\frac{AD}{BD} = \frac{3}{4}$ Find $\frac{BC}{DE}$

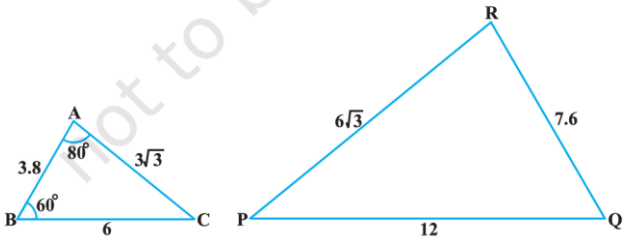
****Q28.**In the given figure, ∠ACB = ∠CDA, AC = 8cm, AD = 3cm, then find BD.



****Q29.** In the given figure DE // BC and $\frac{AD}{BD} = \frac{3}{5}$ if AC = 4.8cm, find the length of AE.

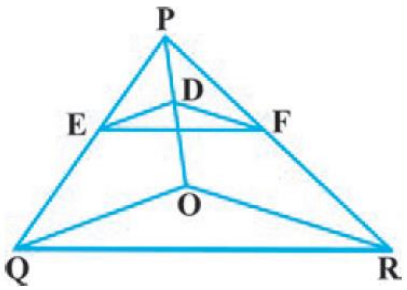


***Q30.** Observe the Figure and then find ∠ P.

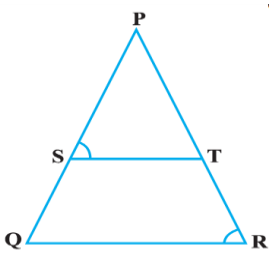


****Q31.** If AD and PM are medians of triangles ABC and PQR, respectively where Δ ABC ~ Δ PQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

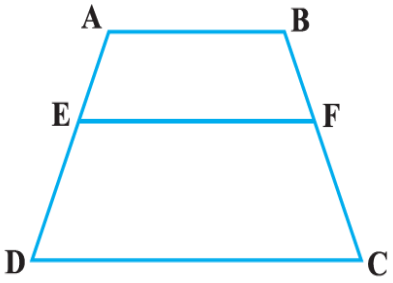
****Q32.** In the given Figure, DE || OQ and DF || OR.
Show that EF || QR.



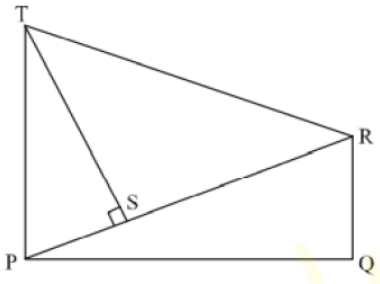
****Q33.** In the given Figure, $\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠ PST = ∠ PRQ. Prove that PQR is an isosceles triangle



***Q34.** ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB. Show that $\frac{AE}{ED} = \frac{BF}{FC}$



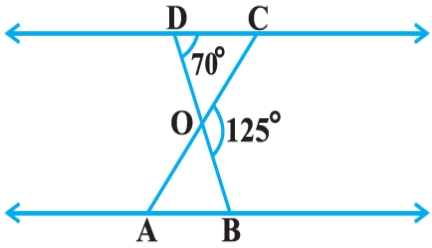
***Q35.** In the given figure, RQ and TP are perpendicular to PQ, also TS perpendicular to PR . Prove that ST.RQ = PS.PQ.



SECTION D

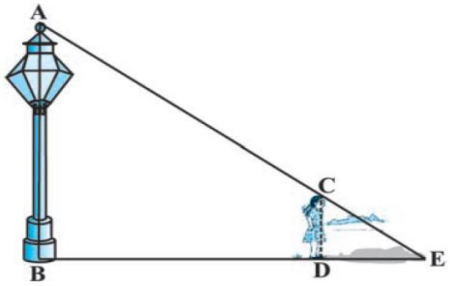
(LONG ANSWER TYPE QUESTIONS 5 MARKS)

****Q36.** In the given figure,Δ ODC ~ Δ OBA, ∠BOC = 125°and ∠ CDO = 70°. Find ∠ DOC, ∠DCO and∠ OAB.

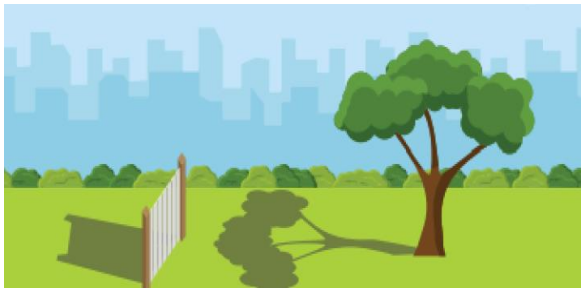


*****Q37.**A vertical pole of length 3 m casts a shadow 2 m long on the ground and at the same time a tower casts a shadow 14 m long. Find the height of the tower.

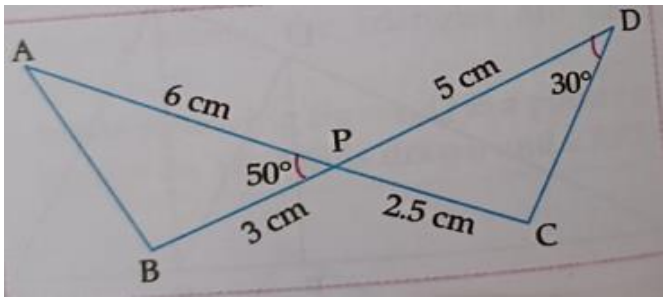
***Q38.A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.



**Q39. Raj wanted to determine the height of a tree on the corner of his block. He knew that a certain fence by the tree was 4 feet tall. At 3 PM, he measured the shadow of the fence to be 2.5 feet tall. Then he measured the tree's shadow to be 11.3 feet. What is the height of the tree?



*Q40. In the given figure, two line segment AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, ∠APB = 50º, and ∠CDP= 30º, Then Find the ∠PBA.

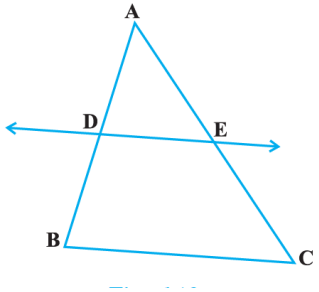


***Q41.A 15m high tower casts a shadow 24m long at a certain time and at the same time, a telephone pole casts a shadow 16m long. Find the height of the telephone pole.

***Q42. Let ABC be a triangle and D and E be two points on side AB such that AD = BE. If DP // BC and EQ // AC, then prove that PQ // AB.

***Q43.State and prove Thales' theorem.

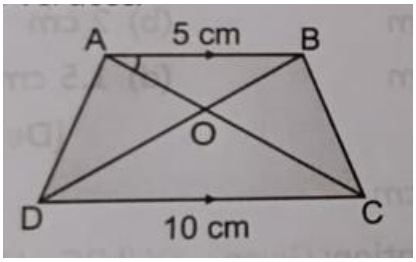
**Q44. If a line intersects sides AB and AC of a Δ ABC at D and E respectively and is parallel to BC, prove that $\frac{AD}{AB} = \frac{AE}{AC}$



***Q45. D is a point on the side BC of a triangle ABC such that ∠ ADC = ∠ BAC. Show that CA^2 = CB.CD.

CASE BASED QUESTIOS

****Q46.** A farmer has a field in the shape of a trapezium, whose map with scale 1cm = 20m, is given below: The field is divided into four parts by joining the opposite vertices.



Based on the above information, answer the questions:

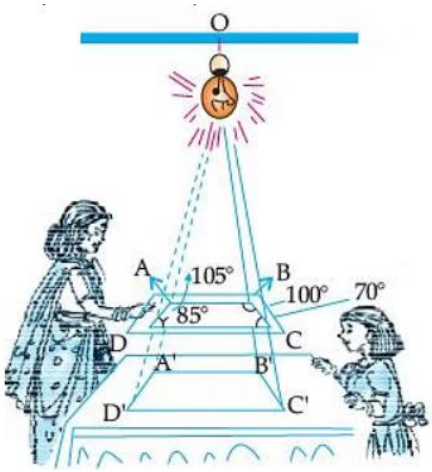
(i) If ABCD is a trapezium in which AB // DC and its diagonals intersect each other at the point O then AO/BO is equal to_____

(ii) In a trapezium ABCD, AB // CD, the diagonal AC and BD intersect at O. If OC = 3 cm, OB = 2 cm and OA = 4 cm then find the length of side OD.

(iii) Prove that the two triangular region AOB and COD are similar.

***Q47**. Read the following text and answer the below questions

Seema placed a light bulb at a point O on the ceiling and directly below it placed a table. Now she put a cardboard of shape ABCD between table and light bulb. Then a shadow of ABCD is casted on the table as $A^1B^1C^1D^1$ (See figure). Quadrilateral $A^1B^1C^1D^1$is in an enlargement of ABCD with scale factor 1:2 also AB = 1.5 cm, BC = 25 cm, CD = 2.4cm and AD = 2.1 cm, ∠ A=105º, ∠B = 100º, ∠X = 70º and ∠ D = 85º

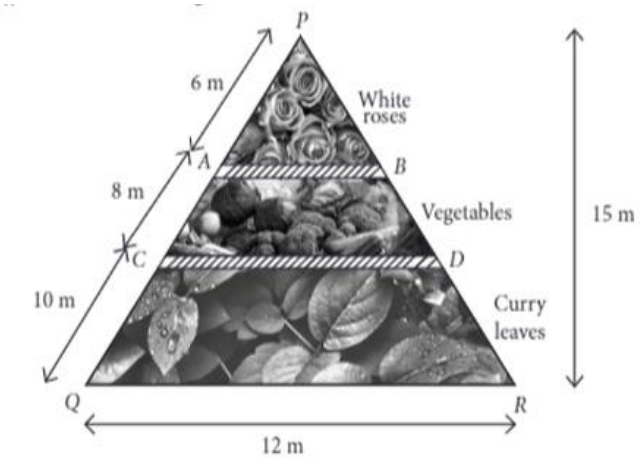


(i) What is the sum of $\angle C^1$and $\angle D^1$

(ii) What is the measure of $\angle A^1$

(iii) What is the length ofA^1B^1

****Q48.** In the backyard of house, Meeta has some empty space in the shape of a ΔPQR. She decided to make it a garden. She divided the whole space into three parts by making boundaries AB and CD using bricks to grow flower ,where AB//CD//QR as shown in figure.



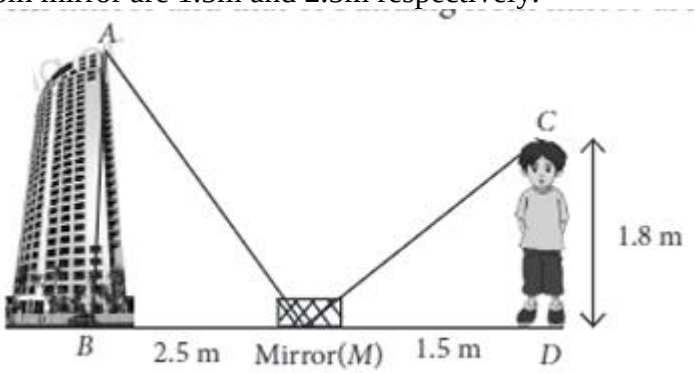
Based on the above information, answer the questions:

(i) What is the length of AB ?

(ii) What is the length of CD ?

(iii) What is the area of whole land?

*****Q49.** Ram is a student of class X, One day his math teacher gave an activity to measure the height of the building. Ram accepted the challenge and places a mirror on ground level to determine the height of building .He is standing at a certain distance so that he can see the top of the building reflected from mirror. Ram's eye level is at 1.8m above the ground. The distance of Ram from the mirror and that of building from mirror are 1.5m and 2.5m respectively.



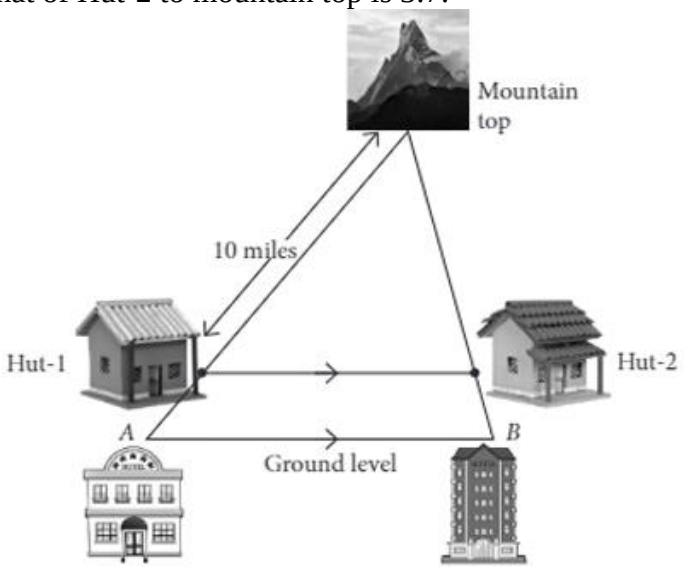
Based on the above information, answer the questions:

(i) Name the triangles which are similar and also mentioned which criterion of similarity is applied here?

(ii) What is the height of the building?

(iii) If Δ ABM and ΔCDA are similar where CD = 6cm, MD = 8 cm and BM = 24cm, then find the length of AB?

****Q50.** Two hotels are at the ground level near to a mountain. On moving a certain distance toward the top of the mountain two huts are situated as shown in the figure. The ratio between the distance from hotel B to Hut-2 and that of Hut-2 to mountain top is 3:7.



Based on the above information, answer the questions:

(i) What is the distance between the hotel A and Hut-1?

(ii) If the horizontal distance between the Hut-1 and Hut-2 is 8miles, then find the distance between two hotels?

(iii) If the distance from mountain top to Hut-1 is 5miles more than that of distance from hotel B to mountain top, then what is the distance between Hut-2 and mountain top?

ANSWER KEY
CHAPTER-6 TRIANGLES
SECTION A

QUESTION	ANSWER	QUESTION	ANSWER	QUESTION	ANSWER
1	D	**6**	D	**11**	A
2	C	**7**	C	**12**	C
3	D	**8**	C	**13**	B
4	C	**9**	A	**14**	B
5	C	**10**	B	**15**	B

SECTION B

QUESTION	ANSWER
16	14 cm.
17	24m
18	3 cm
19	6cm
20	5cm

SECTION C

QUESTION	ANSWER
26	3cm.
27	$-\frac{7}{3}$
28	55/3cm
29	1.8cm
30	$\angle P = 40°$

SECTION D

QUESTION	ANSWER
36	$\angle DOC=55^o, \angle DCO =55^o$ and $\angle OAB = 55^o$
37	21m
38	1.6 m long
39	Height of the tree is 18.08 feet
40	100^o
41	10m

CASE BASED QUESTIOS

46.(i) CO/DO
(ii) 6cm
(iii) Prove

47.(i) 155^o
(ii) 105^o
(iii) 3cm

48. (i) 3m
(ii) 7cm
(iii) $90m^2$

49. (i)- ΔABM and ΔCDM, AA Criterion.
(ii) 3cm
(iii) 18cm

50. (i) 4.29 miles
(ii) 11.43 miles
(iii) 3.5miles

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